1. Aufgabe

a1) $f_k(x) = \dfrac{x^2 - 2k}{x + 3k}$

$$f_k'(x) = \frac{(x+3k)\cdot 2x - (x^2-2k)\cdot 1}{(x+3k)^2} = \frac{2x^2 + 6kx - x^2 + 2k}{(x+3k)^2} = \frac{x^2 + 6kx + 2k}{(x+3k)^2}$$

$$f_1(x) = \frac{x^2-2}{x+3}; \quad f_{-2}(x) = \frac{x^2+4}{x-6}; \quad f_1'(x) = \frac{x^2+6x+2}{(x+3)^2}; \quad f_{-2}'(x) = \frac{x^2-12x-4}{(x-6)^2}$$

a2) $f_k(x) = \dfrac{x^2 - 2k}{(x + 3k)^2}$

$$f_k'(x) = \frac{(x+3k)^2\cdot 2x - (x^2-2k)\cdot 2(x+3k)}{(x+3k)^4} = \frac{(x+3k)[2x^2+6kx-2x^2+4k]}{(x+3k)^4} = \frac{6kx+4k}{(x+3k)^3}$$

$$f_1(x) = \frac{x^2-2}{(x+3)^2}; \quad f_{-2}(x) = \frac{x^2+4}{(x-6)^2}; \quad f_1'(x) = \frac{6x}{(x+3)^3}; \quad f_{-2}'(x) = -\frac{12x}{(x-6)^3}$$

b1) $f_a(x) = \dfrac{ax^2}{ax+2}$

$$f_a'(x) = \frac{(ax+2)\cdot 2ax - ax^2\cdot a}{(ax+2)^2} = \frac{2a^2x^2 + 4ax - a^2x^2}{(ax+2)^2} = \frac{a^2x^2+4ax}{(ax+2)^2}$$

$$f_1(x) = \frac{x^2}{x+2}; \quad f_1'(x) = \frac{x^2+4x}{(x+2)^2}; \quad f_{-2}(x) = \frac{x^2}{x-1}; \quad f_{-2}'(x) = \frac{4x^2-8x}{(-2x+2)^2} = \frac{x^2-2x}{(x-1)^2}$$

b2) $f_a(x) = \dfrac{ax^2}{(ax+2)^2}$

$$f_a'(x) = \frac{(ax+2)^2\cdot 2ax - 2(ax+2)a\cdot ax^2}{(ax+2)^4} = \frac{(ax+2)[2a^2x^2+4ax-2a^2x^2]}{(ax+2)^4} = \frac{4ax}{(ax+2)^3}$$

$$f_1(x) = \frac{x^2}{(x+2)^2}; \quad f_1'(x) = \frac{4x}{(x+2)^3}; \quad f_{-2}(x) = \frac{-x^2}{2(x-1)^2}; \quad f_{-2}'(x) = \frac{-4x^2}{(-2x+2)^3} = \frac{x^2}{2(x-1)^2}$$

c) $f_c(x) = \ln\left(\dfrac{x-c}{x+c}\right)$

$$f_c'(x) = \frac{x+c}{x-c}\cdot\frac{(x+c)\cdot 1 - (x-c)\cdot 1}{(x+c)^2} = \frac{x+c-x+c}{(x+c)(x-c)} = \frac{2c}{(x+c)(x-c)}$$

$$f_1(x) = \ln\left(\frac{x-1}{x+1}\right); \quad f_1'(x) = \frac{2}{(x+1)(x-1)}; \quad f_{-2}(x) = \ln\left(\frac{x+2}{x-2}\right); \quad f_{-2}'(x) = \frac{-4}{(x-2)(x+2)}$$

2 Aufgabe

a) D = IR\{-3}

Nullstellen: f(x) = 0 => $x_{1/2} = \pm\sqrt{k}$
keine Nst. für k < 0
genau 1 Nst. für k = 0: x = 0
für k = 9: x = 3
genau 2 Nst. für k ∈ IR^+\{0; 9}: $x_{1/2} = \pm\sqrt{k}$

$$f_k'(x) = \frac{(x+3)\cdot 2x - (x^2-k)\cdot 1}{(x+3)^2} = \frac{2x^2+6x-x^2+k}{(x+3)^2} = \frac{x^2+6x+k}{(x+3)^2}$$

b) D = IR\{2}

Nullstellen: f(x) = 0 => $x_{1/2} = \pm\sqrt{k}$
keine Nst. für k < 0
genau 1 Nst. für k = 0: x = 0
für k = 4: x = –2
genau 2 Nst. für k ∈ IR^+\{0; 4}: $x_{1/2} = \pm\sqrt{k}$

$$f_k'(x) = \frac{(x-2)\cdot 2x - (x^2-k)\cdot 1}{(x-2)^2} = \frac{2x^2-4x-x^2+k}{(x-2)^2} = \frac{x^2-4x+k}{(x-2)^2}$$

c) D = IR\{-1;2}

Nullstellen: $f_k(x) = 0$ => $x_{1/2} = \pm\sqrt{c}$
keine Nst. für c < 0
genau 1 Nst. für k = 0: x = 0
für k = 1: x = 1
für k = 4: x = 2
genau 2 Nst. für k ∈ IR^+\{0; 9}: $x_{1/2} = \pm\sqrt{c}$

$$f_c'(x) = \frac{(x^2-x-2)\cdot 2x - (2x-1)\cdot(x^2-c)}{(x+1)^2(x-2)^2} = \frac{2x^3-2x^2-4x-2x^3+2cx+x^2-c}{(x+1)^2(x+3)^2} = \frac{-x^2+2(c-2)x-c}{(x+1)^2(x-2)^2}$$

d) D = IR\{–3}

Nullstellen: $f_k(x) = 0$

$$x_{1/2} = \frac{-6\pm\sqrt{36-4k}}{2} = -3\pm\sqrt{9-k}$$

keine Nst. für k > 9
genau 2 Nst. für k < 9: $x_{1/2} = -3\pm\sqrt{9-k}$

$$f_k'(x) = \frac{(x+3)\cdot(2x+6) - (x^2+6x+k)\cdot 1}{(x+3)^2} = \frac{2x^2+12x+18-x^2-6x-k}{(x+3)^2} = \frac{x^2+6x+18-k}{(x+3)^2}$$

e) D = IR\{2}

Nullstellen: $f_a(x) = 0$

$$x_{1/2} = \frac{4\pm\sqrt{16-4a}}{2} = 2\pm\sqrt{4-a}$$

keine Nst. für a > 4
genau 2 Nst. für a < 4: $x_{1/2} = 2\pm\sqrt{4-a}$

$$f_a'(x) = \frac{(x-2)\cdot(2x-4) - (x^2-4x+a)\cdot 1}{(x-2)^2} = \frac{2x^2-8x+8-x^2+4x-a}{(x-2)^2} = \frac{x^2-4x+8-a}{(x-2)^2}$$

f) D = IR\{–2;2}

Nullstellen: $f_k(x) = 0$

$$x_{1/2} = \frac{2k\pm\sqrt{4k^2-4k^2}}{2} = k$$ (besser: bin. Formel)

keine Nst. für k = –2;2
genau 1 Nst. für k ∈ IR\{-2; 2}: x =k

$$f_k'(x) = \frac{(x^2-4)\cdot(2x-2k) - 2x\cdot(x^2-2kx+k^2)}{(x+2)^2(x-2)^2} = \frac{2x^3-2kx^2-8x+8k-2x^3+4kx^2-2k^2x)}{(x+2)^2(x-2)^2} = \frac{2kx^2-2(4+k^2)x+8k}{(x+2)^2(x-2)^2}$$