**Aufgabe I:**

a) Definitionsmenge:
D = IR\{-2}

Grenzverhalten:

- $\lim\limits_{x\to-2^+} f(x) = \lim\limits_{h\to0} f(-2+h) = \lim\limits_{h\to0} \frac{(-2+h)^2-(-2+h)-6}{-2+h+2} =$
$= \lim\limits_{h\to0} \frac{4-4h+h^2+2-h-6}{h} = \lim\limits_{h\to0} \frac{-5h+h^2}{h} = \lim\limits_{h\to0} \frac{h(-5+h)}{h} = \lim\limits_{h\to0}(-5+h) = -5$
- $\lim\limits_{x\to-2^-} f(x) = \lim\limits_{h\to0} f(-2-h) = \lim\limits_{h\to0} \frac{(-2-h)^2-(-2-h)-6}{-2-h+2} =$
$= \lim\limits_{h\to0} \frac{4+4h+h^2+2+h-6}{-h} = \lim\limits_{h\to0} \frac{5h+h^2}{-h} = \lim\limits_{h\to0} \frac{-h(-5-h)}{-h} = \lim\limits_{h\to0}(-5-h) = -5$

Damit: $\lim\limits_{x\to-2} f(x) = -5$

b) Definitionsmenge:
D = IR\{4}

Grenzverhalten:

- $\lim\limits_{x\to4^+} f(x) = \lim\limits_{h\to0} f(4+h) = \lim\limits_{h\to0} \frac{(4+h)^2-7(4+h)+12}{4+h-4} =$
$= \lim\limits_{h\to0} \frac{16+8h+h^2-28-7h+12}{h} = \lim\limits_{h\to0} \frac{h+h^2}{h} = \lim\limits_{h\to0} \frac{h(1+h)}{h} = \lim\limits_{h\to0}(1+h) = 1$
- $\lim\limits_{x\to4^-} f(x) = \lim\limits_{h\to0} f(4-h) = \lim\limits_{h\to0} \frac{(4-h)^2-7(4-h)+12}{4-h-4} =$
$= \lim\limits_{h\to0} \frac{16-8h+h^2-28+7h+12}{-h} = \lim\limits_{h\to0} \frac{-h+h^2}{-h} = \lim\limits_{h\to0} \frac{-h(1-h)}{-h} = \lim\limits_{h\to0}(1-h) = 1$

Damit: $\lim\limits_{x\to3} f(x) = 1$

c) Definitionsmenge:
D = IR\{3}

Grenzverhalten:

- $\lim\limits_{x\to3^+} f(x) = \lim\limits_{h\to0} f(3+h) = \lim\limits_{h\to0} \frac{(3+h)^2-7(3+h)+12}{3+h-3} =$
$= \lim\limits_{h\to0} \frac{9+6h+h^2-21-7h+12}{h} = \lim\limits_{h\to0} \frac{-h+h^2}{h} = \lim\limits_{h\to0} \frac{h(-1+h)}{h} = \lim\limits_{h\to0}(-1+h) = -1$
- $\lim\limits_{x\to3^-} f(x) = \lim\limits_{h\to0} f(3-h) = \lim\limits_{h\to0} \frac{(3-h)^2-7(3-h)+12}{3-h-3} =$
$= \lim\limits_{h\to0} \frac{9-6h+h^2-21+7h+12}{-h} = \lim\limits_{h\to0} \frac{h+h^2}{-h} = \lim\limits_{h\to0} \frac{-h(-1-h)}{-h} = \lim\limits_{h\to0}(-1-h) = -1$

Damit: $\lim\limits_{x\to3} f(x) = -1$

d) Definitionsmenge:
D = IR\{-3}

Grenzverhalten:

- $\lim\limits_{x \to -3^+} f(x) = \lim\limits_{h \to 0} f(-3+h) = \lim\limits_{h \to 0} \dfrac{(-3+h)^2 - 2(-3+h) - 15}{-3+h+3} =$

  $= \lim\limits_{h \to 0} \dfrac{9-6h+h^2+6-2h-15}{h} = \lim\limits_{h \to 0} \dfrac{-8h+h^2}{h} = \lim\limits_{h \to 0} \dfrac{h(-8+h)}{h} = \lim\limits_{h \to 0}(-8+h) = -8$

- $\lim\limits_{x \to -3^-} f(x) = \lim\limits_{h \to 0} f(-3-h) = \lim\limits_{h \to 0} \dfrac{(-3-h)^2 - 2(-3-h) - 15}{-3-h+3} =$

  $= \lim\limits_{h \to 0} \dfrac{9+6h+h^2+6+2h-15}{-h} = \lim\limits_{h \to 0} \dfrac{8h+h^2}{-h} = \lim\limits_{h \to 0} \dfrac{-h(-8-h)}{-h} = \lim\limits_{h \to 0}(-8-h) = -8$

Damit: $\lim\limits_{x \to -3} f(x) = -8$

e) Definitionsmenge:
D = IR\{5}

Grenzverhalten:

- $\lim\limits_{x \to 5^+} f(x) = \lim\limits_{h \to 0} f(5+h) = \lim\limits_{h \to 0} \dfrac{(5+h)^2 - 2(5+h) - 15}{5+h-5} =$

  $= \lim\limits_{h \to 0} \dfrac{25+10h+h^2-10-2h-15}{h} = \lim\limits_{h \to 0} \dfrac{8h+h^2}{h} = \lim\limits_{h \to 0} \dfrac{h(8+h)}{h} = \lim\limits_{h \to 0}(8+h) = 8$

- $\lim\limits_{x \to 5^-} f(x) = \lim\limits_{h \to 0} f(5-h) = \lim\limits_{h \to 0} \dfrac{(5-h)^2 - 2(5-h) - 15}{5-h-5} =$

  $= \lim\limits_{h \to 0} \dfrac{25-10h+h^2-10+2h-15}{-h} = \lim\limits_{h \to 0} \dfrac{-8h+h^2}{-h} = \lim\limits_{h \to 0} \dfrac{-h(8-h)}{-h} = \lim\limits_{h \to 0}(8-h) = 8$

Damit: $\lim\limits_{x \to 5} f(x) = 8$

f) Definitionsmenge:
D = IR\{3}

Grenzverhalten:

- $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{h \to 0} f(3+h) = \lim\limits_{h \to 0} \dfrac{(3+h)^2 - (3+h) - 6}{3-h-3} =$

  $= \lim\limits_{h \to 0} \dfrac{9+6h+h^2-3-h-6}{h} == \lim\limits_{h \to 0} \dfrac{5h+h^2}{h} = \lim\limits_{h \to 0} \dfrac{h(5+h)}{h} = \lim\limits_{h \to 0}(5+h) = 5$

- $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{h \to 0} f(3-h) = \lim\limits_{h \to 0} \dfrac{(3-h)^2 - (3-h) - 6}{3-h-3} =$

  $= \lim\limits_{h \to 0} \dfrac{9-6h+h^2-3+h-6}{-h} == \lim\limits_{h \to 0} \dfrac{-5h+h^2}{-h} = \lim\limits_{h \to 0} \dfrac{-h(5-h)}{-h} = \lim\limits_{h \to 0}(5-h) = 5$

Damit: $\lim\limits_{x \to 3} f(x) = 5$

**Aufgabe II:**

a) Definitionsmenge:

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x_1 = 0$

$x_2 = -2$ D = IR\{–2; 0}

Grenzverhalten: (Ränder sind: ±∞; –2 und 0)

▶ $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \frac{x^2 - x - 6}{x^2 + 2x} = \lim\limits_{x\to\pm\infty} \frac{1 - \frac{1}{x} - \frac{6}{x^2}}{1 + \frac{2}{x}} = 1$ horizontale Asymptote: y =1

▶ x→ –2

• $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{h\to 0} f(-2-h) = \lim\limits_{h\to 0} \frac{(-2-h)^2 - (-2-h) - 6}{(-2-h)^2 + 2(-2-h)} = \lim\limits_{h\to 0} \frac{4 + 4h + h^2 + 2 + h - 6}{4 + 4h + h^2 - 4 - 2h} =$

$= \lim\limits_{h\to 0} \frac{5h + h^2}{2h + h^2} = \lim\limits_{h\to 0} \frac{5 + h}{2 + h} = \frac{5}{2}$

• $\lim\limits_{x\to -2^+} f(x) = \lim\limits_{h\to 0} f(-2+h) = \lim\limits_{h\to 0} \frac{(-2+h)^2 - (-2+h) - 6}{(-2+h)^2 + 2(-2+h)} = \lim\limits_{h\to 0} \frac{4 - 4h + h^2 + 2 - h - 6}{4 - 4h + h^2 - 4 + 2h} =$

$= \lim\limits_{h\to 0} \frac{-5h + h^2}{-2h + h^2} = \lim\limits_{h\to 0} \frac{5 - h}{2 - h} = \frac{5}{2}$

• Also: $\lim\limits_{x\to -2} f(x) = 2{,}5$ "Loch": L(–2|2,5)

▶ x→ 0

• $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(-h) = \lim\limits_{h\to 0} \frac{(-h)^2 - (-h) - 6}{(-h)^2 + 2(-h)} =$ Beachte: f(0 – h) = f(–h)

$= \lim\limits_{h\to 0} \frac{h^2 + h - 6}{h^2 - 2h} = \lim\limits_{h\to 0} \frac{h^2 + h - 6}{h(h-2)} = +\infty$

• $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} f(h) = \lim\limits_{h\to 0} \frac{h^2 - h - 6}{h^2 + 2h} = \lim\limits_{h\to 0} \frac{h^2 + h - 6}{h^2 + 2h} =$

$= \lim\limits_{h\to 0} \frac{h^2 + h - 6}{h(h+2)} = -\infty$

• Also: Vertikale Asymptote: x = 0

b) Definitionsmenge:

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x_1 = 4$

$x_2 = -2$ $D = IR\backslash\{-2; 4\}$

Grenzverhalten: (Ränder sind: $\pm\infty$; $-2$ und $4$)

▶ $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \frac{x^2-7x+12}{x^2-2x-8} = \lim\limits_{x\to\pm\infty} \frac{1-\frac{7}{x}+\frac{12}{x^2}}{1-\frac{2}{x}-\frac{8}{x^2}} = 1$ horizontale Asymptote: $y = 1$

▶ $x \to -2$

- $\lim\limits_{x\to-2^-} f(x) = \lim\limits_{h\to0} f(-2-h) = \lim\limits_{h\to0} \frac{(-2-h)^2-7(-2-h)+12}{(-2-h)^2-2(-2-h)-8} =$

  $= \lim\limits_{h\to0} \frac{4+4h+h^2+14+7h+12}{4+4h+h^2+4+2h-8} = \lim\limits_{h\to0} \frac{11h+h^2+30}{6h+h^2} = \lim\limits_{h\to0} \frac{11h+h^2+30}{h(6+h)} = +\infty$

- $\lim\limits_{x\to-2^+} f(x) = \lim\limits_{h\to0} f(-2+h) = \lim\limits_{h\to0} \frac{(-2+h)^2-7(-2+h)+12}{(-2+h)^2-2(-2+h)-8} =$

  $= \lim\limits_{h\to0} \frac{4-4h+h^2+14-7h+12}{4-4h+h^2+4-2h-8} = \lim\limits_{h\to0} \frac{-11h+h^2+30}{-6h+h^2} = \lim\limits_{h\to0} \frac{-11h+h^2+30}{h(-6+h)} = -\infty$

- Also: Vertikale Asymptote: $x = -2$

▶ $x \to 4$

- $\lim\limits_{x\to-4^-} f(x) = \lim\limits_{h\to0} f(4-h) = \lim\limits_{h\to0} \frac{(4-h)^2-7(4-h)+12}{(4-h)^2-2(4-h)-8} =$

  $= \lim\limits_{h\to0} \frac{16-8h+h^2-28+7h+12}{16-8h+h^2-8+2h-8} = \lim\limits_{h\to0} \frac{-h+h^2}{-6h+h^2} = \lim\limits_{h\to0} \frac{-h(1-h)}{-h(6-h)} = \lim\limits_{h\to0} \frac{(1-h)}{(6-h)} = \frac{1}{6}$

- $\lim\limits_{x\to-4^+} f(x) = \lim\limits_{h\to0} f(4+h) = \lim\limits_{h\to0} \frac{(4+h)^2-7(4+h)+12}{(4+h)^2-2(4+h)-8} =$

  $= \lim\limits_{h\to0} \frac{16+8h+h^2-28-7h+12}{16+8h+h^2-8-2h-8} = \lim\limits_{h\to0} \frac{h+h^2}{6h+h^2} = \lim\limits_{h\to0} \frac{h(1+h)}{h(6+h)} = \lim\limits_{h\to0} \frac{1+h}{6+h} = \frac{1}{6}$

- Also: $\lim\limits_{x\to4} f(x) = \frac{1}{6}$ "Loch": $L(-2|\frac{1}{6})$

c) Definitionsmenge:

– 2x – 6 = 0

–2x = 6

x = –3 D = IR\{–3}

Grenzverhalten: (Ränder sind: ±∞ und –3)

▶ $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \frac{4x+12}{-2x-6} = \lim\limits_{x\to\pm\infty} \frac{4+\frac{12}{x}}{-2-\frac{6}{x}} = -2$ horizontale Asymptote: y =–2

▶ x→ –3

- $\lim\limits_{x\to -3^-} f(x) = \lim\limits_{h\to 0} f(-3-h) = \lim\limits_{h\to 0} \frac{+4(-3-h)+12}{-2(-3-h)-6} = \lim\limits_{h\to 0} \frac{-12-4h+12}{6+2h-6}$

  $= \lim\limits_{h\to 0} \frac{-4h}{2h} = \lim\limits_{h\to 0}(-2) = -2$

- $\lim\limits_{x\to -3^+} f(x) = \lim\limits_{h\to 0} f(-3+h) = \lim\limits_{h\to 0} \frac{+4(-3+h)+12}{-2(-3+h)-6} = \lim\limits_{h\to 0} \frac{-12+4h+12}{6-2h-6}$

  $= \lim\limits_{h\to 0} \frac{4h}{-2h} = \lim\limits_{h\to 0}(-2) = -2$

- Also: $\lim\limits_{x\to -3} f(x) = -2$ "Loch": L(–3|–2)

**Aufgabe II:**

1. f(x) = $\dfrac{x^3+4x^2-7x-10}{2x^3+16}$

a) Definitionsmenge:

$2x^3 + 16 = 0$

$x^3 = -8$

$x = -2$ D = IR\{–2}

b) SP mit x-Achse

Bed.: f(x) = 0

$x^3+4x^2-7x-10=0$ $x_1 = -1$ (erraten)

$(x^3+4x^2-7x-10):(x+1) = x^2 + 3x - 10 = (x-2)(x+5)$ (Polynomdivision, Vieta)

$x_2 = 2$; $x_3 = -5$

$N_1(-1|0)$ $N_2(2|0)$ $N_3(-5|0)$

SP mit y-Achse

$$f(0) = \frac{0^3+4\cdot 0^2-7\cdot 0-10}{2\cdot 0^3+16} = \frac{-10}{16} = -\frac{5}{4} = -1{,}25$$

$S_y(0|-1{,}25)$

d) Grenzverhalten: (Ränder sind: ±∞ und –2)

► $$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x^3+4x^2-7x-10}{2x^3+16} = \lim_{x\to\pm\infty} \frac{1+\frac{4}{x}-\frac{7}{x^2}-\frac{10}{x^3}}{2-\frac{16}{x^3}} = \frac{1}{2}$$

horizontale Asymptote: $y = \frac{1}{2}$

► $x \to -2$

• $$\lim_{x\to -2^-} f(x) = \lim_{h\to 0} f(-2-h) = \lim_{h\to 0} \frac{(-2-h)^3+4(-2-h)^2-7(-2-h)-10}{2(-2-h)^3+16} =$$

$$= \lim_{h\to 0} \frac{-8-12h-6h^2-h^3+4(4+4h+h^2)+14+7h-10}{2(-8-12h-6h^2-h^3)+16} =$$

$$= \lim_{h\to 0} \frac{-8-12h-6h^2-h^3+16+16h+4h^2+14+7h-10}{-16-24h-12h^2-2h^3+16} =$$

$$= \lim_{h\to 0} \frac{12+11h-3h^2}{-24h-12h^2-2h^3} = \lim_{h\to 0} \frac{\overset{+}{12+11h-3h^2}}{\underset{-\,+}{h}\underset{-}{(-24-12h-2h^2)}} = -\infty$$

• $$\lim_{x\to -2^+} f(x) = \lim_{h\to 0} f(-2+h) = \lim_{h\to 0} \frac{(-2+h)^3+4(-2+h)^2-7(-2+h)-10}{2(-2+h)^3+16} =$$

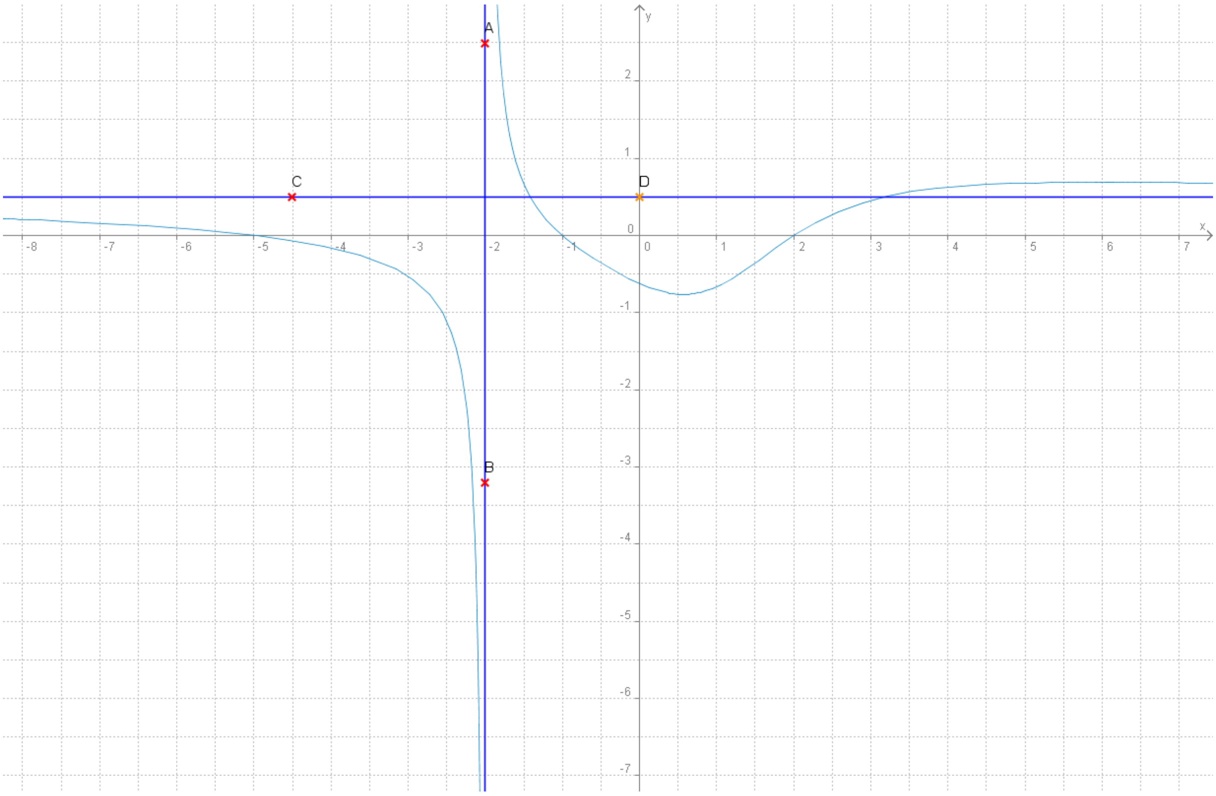
$$= \lim_{h\to 0} \frac{-8+12h-6h^2+h^3+4(4-4h+h^2)+14-7h-10}{2(-8+12h-6h^2+h^3)+16} =$$

$$= \lim_{h\to 0} \frac{-8+12h-6h^2+h^3+16-16h+4h^2+14-7h-10}{-16+24h-12h^2+2h^3+16} =$$

$$= \lim_{h\to 0} \frac{12-11h-2h^2+h^3}{24h-12h^2+2h^3} = \lim_{h\to 0} \frac{12-11h-2h^2+h^3}{2h(12-6h+h^2)} = +\infty$$

- Also: Vertikale Asymptote: x = -2

e)



2. $f(x) = \frac{4x^2 - 9}{x^3}$

a) <u>Definitionsmenge:</u>
$x^3 = 0$
$x = 0$ $D = IR\setminus\{0\}$

b) <u>SP mit x-Achse</u>
Bed.: $f(x) = 0$
$4x^2 - 9 = 0$
$x^2 - 9 = 9$
$x^2 = \frac{9}{4}$
$x = \pm\frac{3}{2}$
$x_1 = -1{,}5$
$x_2 = +1{,}5$
$N_1(-1{,}5|0)$ $N_2(1{,}5|0)$

<u>SP mit y-Achse</u>
Keiner vorhanden, da $0 \notin D$

c) $f(-x) = \frac{4(-x)^2 - 9}{(-x)^3} = \frac{4x^2 - 9}{-x^3} = -\frac{4x^2 - 9}{x^3} = -f(x)$
Graph punktsymmetrisch zum Ursprung

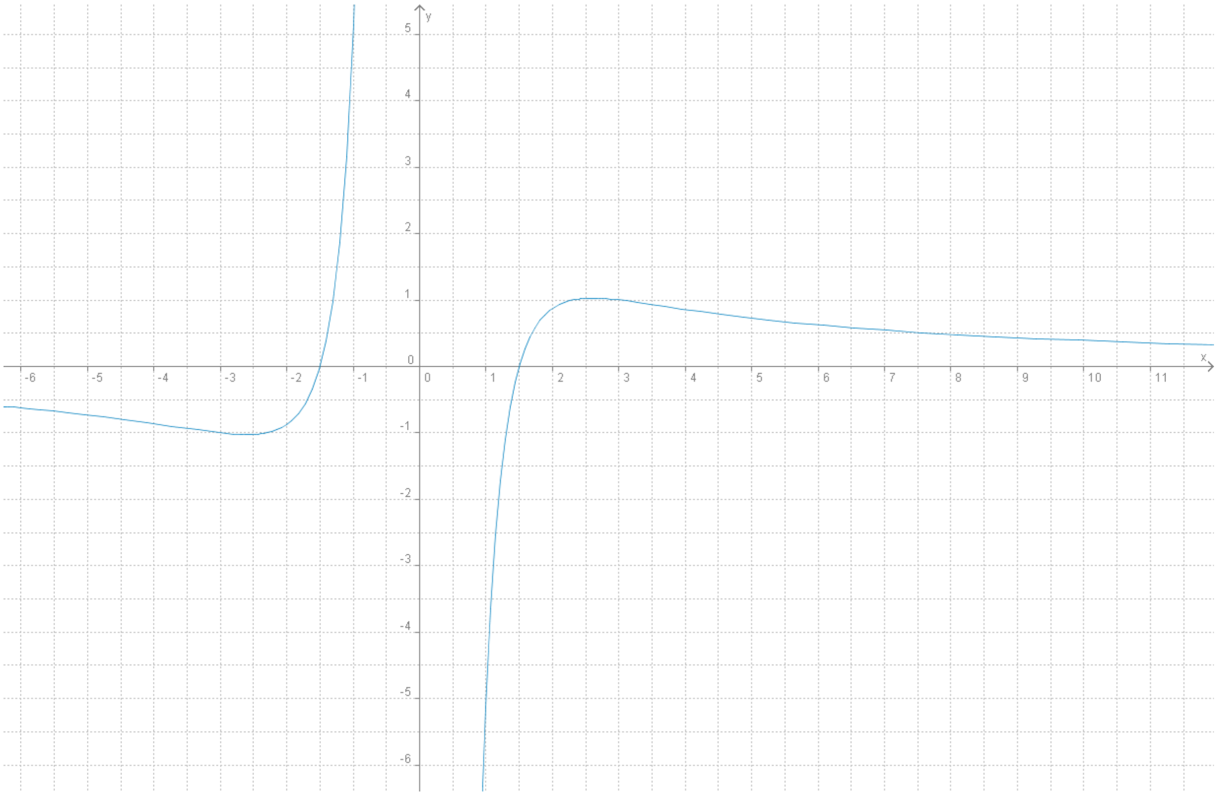
d) <u>Grenzverhalten:</u> (Ränder sind: ±∞ und 0)

▶ $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} = \frac{4x^2 - 9}{x^3} = \lim\limits_{x\to\pm\infty} \frac{\frac{4}{x} - \frac{9}{x^3}}{1} = 0$
horizontale Asymptote: y = 0

▶ $x \to 0$

- $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(-h) = \lim\limits_{h\to 0} \frac{4(-h)^2 - 9}{(-h)^3} = \lim\limits_{h\to 0} \frac{4h^2 - 9}{-h^3} = +\infty$
- $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(-h) = \lim\limits_{h\to 0} \frac{4(-h)^2 - 9}{(-h)^3} = \lim\limits_{h\to 0} \frac{4h^2 - 9}{h^3} = -\infty$
- Also: Vertikale Asymptote: x = 0

e)



3. f(x) = $\dfrac{x^2+x-12}{6x-2x^2}$

a) Definitionsmenge:
6x – 2x² = 0
2x(3 – x) = 0
$x_1 = 0$; $x_2 = 3$ D = IR\{0; 3}

b) SP mit x-Achse
Bed.: f(x) = 0
x² + x – 12 = 0
(x – 3) (x + 4) = 0
($x_1 = 3 \notin$ D);
$x_2 = -4$ $N_1(-4|0)$

SP mit y-Achse
Keiner vorhanden, da $0 \notin$ D

d) Grenzverhalten: (Ränder sind: ±∞; 0 und 3)

▶ $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2+x-12}{6x-2x^2} = \lim\limits_{x\to\pm\infty} \dfrac{1+\frac{1}{x}-\frac{12}{x^2}}{\frac{6}{x}-2} = -\dfrac{1}{2}$ horizontale Asymptote: y = $-\dfrac{1}{2}$

▶ x→ 0

• $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(-h) = \lim\limits_{h\to 0} \dfrac{(-h)^2+(-h)-12}{6(-h)-2(-h)^2} =$

$= \lim\limits_{h\to 0} \dfrac{h^2-h-12}{-6h-2h^2} = \lim\limits_{h\to 0} \dfrac{h^2-h-12}{h(-6-2h)} = +\infty$

• $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} f(h) = \lim\limits_{h\to 0} \dfrac{h^2+h-12}{6h-2h^2} = \lim\limits_{h\to 0} \dfrac{h^2+h-12}{h(6-2h)} = -\infty$

• Also: Vertikale Asymptote: x = 0

▶ x→ 4

• $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{h\to 0} f(3-h) = \lim\limits_{h\to 0} \dfrac{(3-h)^2+(3-h)-12}{6(3-h)-2(3-h)^2} =$

$= \lim\limits_{h\to 0} \dfrac{9-6h+h^2+3-h-12}{18-6h-18+12h-2h^2} = \lim\limits_{h\to 0} \dfrac{-5h+h^2}{6h-2h^2} = \lim\limits_{h\to 0} \dfrac{h(5-h)}{h(6-2h)} = \lim\limits_{h\to 0} \dfrac{(5-h)}{(6-2h)} = \dfrac{5}{6}$

• $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{h\to 0} f(3+h) = \lim\limits_{h\to 0} \dfrac{(3+h)^2+(3+h)-12}{6(3+h)-2(3+h)^2} =$

$= \lim\limits_{h\to 0} \dfrac{9+6h+h^2+3+h-12}{18+6h-18-12h-2h^2} = \lim\limits_{h\to 0} \dfrac{7h+h^2}{-6h-2h^2} = \lim\limits_{h\to 0} \dfrac{h(7-h)}{h(-6-2h)} = \lim\limits_{h\to 0} \dfrac{(7-h)}{(-6-2h)} = -\dfrac{7}{6}$

• Also: $\lim\limits_{x\to 3} f(x) = -\dfrac{7}{6}$ "Loch": $L(3|-\dfrac{7}{6})$

e)